

A HIGH SCHOOLER'S GUIDE TO



# THE MATH SKILLS BOOKLET

All the essential skills for college readiness and beyond.

$$\alpha + \beta$$



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## How to Use This Booklet

1. Follow the Order: The skills are arranged from foundational to advanced. Start from the beginning and move forward for best results.

2. Use the “Additional Resources” section:

-  Watch a short lesson that explains this skill step by step.
-  Download or practice more questions related to the skill.
-  Play an interactive math game to practice in a fun way!



3. Do the “MATH SKILLS TEST” at the end of each chapter.

4.  Check Your Work

At the end of each chapter, you'll find links to Answer Keys for every worksheet or skill page. These are downloadable PDF files to help you review and correct your answers.



 *Click or tap the icons to open the links. All resources are chosen to help you practice, play, and understand math better.*

**This booklet is dedicated to every student who thought they couldn't do math.  
This book is for you.**

## Structure of the Booklet

### ➤ CHAPTER 1: Arithmetic & Number Sense

#### ❖ PART 1: Number System and Arithmetic Operations

- 1.1 Understanding real and complex numbers
- 1.2 Operations with integers
- 1.3 Operations with rational numbers
- 1.4 Order of operations {PEMDAS}
- 1.5 Converting fractions, decimals & percentages
- 1.6 Scientific notation and standard form
- 1.7 Operations with scientific notation

#### ❖ PART 2: Basic Algebraic Thinking

- 1.8 Understanding Variables and Constants
- 1.9 Simplifying Expressions
- 1.10 Solving One-Step Equations
- 1.11 Solving One-Step Inequalities
- 1.12 Solving Two-Steps Equations
- 1.13 Solving Two-Steps Inequalities

### ➤ CHAPTER 2: Algebra Skills

#### ❖ PART 1: Factoring

- 2.1 Greatest Common Factor (GCF)
- 2.2 Factoring by Grouping
- 2.3 Factoring Trinomials ( $x^2+bx+c$ )
- 2.4 Factoring Trinomials ( $ax^2+bx+c$ )
- 2.5 Difference of squares
- 2.6 Perfect square trinomials
- 2.7 Sum and difference of cubes
- 2.8 Completing the Square

#### ❖ PART 2: Understanding Polynomials

- 2.9 Solving Equations by Factoring
- 2.10 Solving Quadratic Equations Using the Quadratic Formula
- 2.11 Understanding polynomial terminology
- 2.12 Adding and subtracting polynomials
- 2.13 Laws of Exponents
- 2.14 Multiplying Polynomials
- 2.15 Dividing Polynomials
- 2.16 Synthetic division

# CHAPTER 1

## Essential Prerequisites

 Learning Objectives: By the end of this chapter, you will be able to:

- ❖ Master all four operations with integers and rational numbers.
- ❖ Apply order of operations correctly in complex expressions.
- ❖ Convert fluently between fractions, decimals, and percentages
- ❖ Use scientific notation for large and small numbers.
- ❖ Understand variables and constants as building blocks of algebra
- ❖ Simplify basic algebraic expressions.

# Mathematical Vocabulary:

- Real Numbers: All numbers on the number line, including rational and irrational numbers.
- Complex Numbers: Numbers that have a real part and an imaginary part.
- Integer: Whole numbers and their negatives.
- Rational Number: Any number that can be written as a fraction  $\frac{a}{b}$ .
- Irrational Number: A number that cannot be written as a simple fraction.
- Natural Numbers: Counting numbers.
- Whole Numbers: Natural numbers and zero.
- Operation: A mathematical process (addition, subtraction, multiplication, division).
- Sum: The result of addition.
- Difference: The result of subtraction.
- Product: The result of multiplication.
- Quotient: The result of division.
- Order of Operations: The rules for the order in which calculations are done.
- Decimal: A number with a decimal point, representing a value less than one.
- Percentage: A way to express a number as a part of 100.
- Scientific Notation: A way to write very large or very small numbers using powers of 10.
- Standard Form: The usual way of writing numbers.
- Variable: A symbol (usually a letter) that represents an unknown value.
- Constant: A fixed value that does not change.
- Expression: A combination of numbers, variables, and operations.
- Equation: A mathematical statement that two expressions are equal.
- Inequality: A statement that compares two values using  $<$ ,  $>$ ,  $\leq$ , or  $\geq$ .
- Solution: A value that makes an equation or inequality true.
- Simplify: To write an expression in its simplest form.
- Substitute: To replace a variable with a number.

**SKILL #05**

CODE: CNV.1

# Converting Fractions, Decimals & Percentages

**Core Concept**

Fractions, decimals, and percentages are just different ways to show parts of a whole. Being able to convert between them helps you work with money, data, and real-life problems easily!

**The Big Picture**

- Fraction: Parts out of a whole ( $\frac{3}{4}$ )
- Decimal: Based on powers of 10 (0.75)
- Percentage: Parts out of 100 (75%)

**FRACTION TO DECIMAL**

Method: Divide the numerator by the denominator

Examples:

- $\frac{1}{2} = 1 \div 2 = 0.5$
- $\frac{3}{4} = 3 \div 4 = 0.75$
- $\frac{1}{8} = 1 \div 8 = 0.125$
- $\frac{2}{5} = 2 \div 5 = 0.4$

Pro Tip: Use long division or a calculator

**DECIMAL TO FRACTION**

1. Write the decimal as a fraction with denominator
2. Simplify by finding the GCD

Examples:

- $0.5 = \frac{5}{10} = \frac{1}{2}$
- $0.75 = \frac{75}{100} = \frac{3}{4}$
- $0.125 = \frac{125}{1000} = \frac{1}{8}$
- $0.6 = \frac{6}{10} = \frac{3}{5}$

**FRACTION TO PERCENTAGE**

Method: Convert to decimal first, then multiply by 100

Examples:

- $\frac{1}{2} = 0.5 \times 100 = 50\%$
- $\frac{3}{4} = 0.75 \times 100 = 75\%$
- $\frac{1}{8} = 0.125 \times 100 = 12.5\%$
- $\frac{2}{5} = 0.4 \times 100 = 40\%$

Shortcut: Multiply fraction by 100%  $\rightarrow \frac{1}{4} \times 100\% = 25\%$

**PERCENTAGE TO FRACTION**

Method: Write as fraction over 100, then simplify

Examples:

- $50\% = \frac{50}{100} = \frac{1}{2}$
- $75\% = \frac{75}{100} = \frac{3}{4}$
- $25\% = \frac{25}{100} = \frac{1}{4}$
- $60\% = \frac{60}{100} = \frac{3}{5}$

## SKILL #05

CODE: CNV.1

# Converting Fractions, Decimals & Percentages

## DECIMAL TO PERCENTAGE



Method: Multiply by 100 and add % symbol

Examples:

- $0.5 = 0.5 \times 100 = 50\%$
- $0.75 = 0.75 \times 100 = 75\%$
- $0.125 = 0.125 \times 100 = 12.5\%$
- $1.25 = 1.25 \times 100 = 125\%$

Memory Trick: Move decimal point 2 places right

## PERCENTAGE TO FRACTION



Method: Divide by 100 (move decimal point 2 places left)

Examples:

- $50\% = 50 \div 100 = 0.5$
- $75\% = 75 \div 100 = 0.75$
- $12.5\% = 12.5 \div 100 = 0.125$
- $125\% = 125 \div 100 = 1.25$

## ESSENTIAL EQUIVALENTS TO MEMORIZE

| Fraction        | Decimal  | Percentage |
|-----------------|----------|------------|
| $\frac{1}{2}$   | 0.5      | 50%        |
| $\frac{1}{4}$   | 0.25     | 25%        |
| $\frac{3}{4}$   | 0.75     | 75%        |
| $\frac{1}{3}$   | 0.333... | 33.33...%  |
| $\frac{2}{3}$   | 0.666... | 66.66...%  |
| $\frac{1}{5}$   | 0.2      | 20%        |
| $\frac{4}{5}$   | 0.8      | 80%        |
| $\frac{1}{8}$   | 0.125    | 12.5%      |
| $\frac{1}{10}$  | 0.1      | 10%        |
| $\frac{1}{100}$ | 0.01     | 1%         |

## SPECIAL CASES

Repeating Decimals:

decimals that repeat forever

If the simplified denominator has any factor other than 2 or 5, the decimal will repeat.

- $\frac{1}{3} = 0.333...$
- $\frac{1}{6} = 0.1666...$
- $\frac{2}{7} = 0.2857142857 ...$

Percentages Over 100%

- $\frac{5}{4} = 1.25 = 125\%$
- $\frac{3}{2} = 1.5 = 150\%$

Percentages Less Than 1%

- $\frac{1}{200} = 0.005 = 0.05\%$
- $\frac{1}{1000} = 0.001 = 0.1\%$

## Common Mistakes to Avoid

- ✗ Forgetting to simplify fractions
- ✗ Moving the decimal point the wrong way
- ✗ Confusing 0.5 with 5% (should be 50%)
- ✗ Treating percentages as decimals without converting

## Additional Resources



NAME: .....

DATE: ...../...../.....

### SECTION A: NUMBER SYSTEMS & OPERATIONS

QUESTION 1: Which of the following is an irrational number?

- a) 0.25                              b)  $\sqrt{9}$                               c)  $\sqrt{7}$                               d)  $\frac{3}{4}$

QUESTION 2: Calculate:  $(-8) + 15 - (-3)$

- a) 4                                      b) 10                                      c) 20                                      d) -26

QUESTION 3: What is  $(-6) \times (-4)$ ?

- a) -24                                      b) 24                                      c) -10                                      d) 10

QUESTION 4: Simplify:  $(-20) \div (-4)$

- a) -5                                      b) 5                                      c) -16                                      d) 16

QUESTION 5: Calculate:  $\frac{1}{3} + \frac{1}{6}$

- a)  $\frac{1}{2}$                                       b)  $\frac{2}{9}$                                       c)  $\frac{1}{9}$                                       d)  $\frac{2}{6}$

QUESTION 6: What is  $\frac{3}{4} - \frac{1}{8}$ ?

- a)  $\frac{2}{4}$                                       b)  $\frac{5}{8}$                                       c)  $\frac{1}{2}$                                       d)  $\frac{3}{8}$

QUESTION 7: Calculate:  $\frac{2}{3} \times \frac{3}{8}$ . Write your answer in simplest form.

- a)  $\frac{1}{4}$                                       b)  $\frac{6}{24}$                                       c)  $\frac{5}{11}$                                       d)  $\frac{1}{2}$

QUESTION 8: What is  $\frac{4}{5} \div \frac{2}{3}$ ? Write your answer in simplest form.

- a)  $\frac{8}{15}$                                       b)  $\frac{6}{5}$                                       c)  $\frac{2}{5}$                                       d)  $\frac{12}{10}$

QUESTION 9: Evaluate:  $3 + 4 \times 2 - 1$

- a) 13                                      b) 10                                      c) 14                                      d) 6

QUESTION 10: Calculate:  $2^3 + 5 \times (6 - 4)$

- a) 18                                      b) 26                                      c) 38                                      d) 22



**SKILL #17**

CODE: FT.2

**Factoring Trinomials ( $ax^2 + bx + c$ )****Core Concept**

Factoring trinomials like  $(ax^2 + bx + c)$  where  $a \neq 1$  requires finding two binomials whose product matches the original. Unlike Skill 16, the coefficient of  $x^2$  is no longer 1, adding complexity.

**When to use it?**

- When the expression has three terms:  $ax^2 + bx + c$
- When factoring trinomials with  $a > 1$
- When solving quadratic equations

**Steps to Solve****MORE EXAMPLES**

Example: Factor  $2x^2 + 7x + 3$ :

STEP 1:  $a = 2, b = 7, c = 3$  --> we can't use direct factoring (skill 16)

STEP 2: multiply a by c -->  $2(3) = 6$

STEP 3: Find the two numbers when multiplied you have 6 and when added you have 7.

STEP 4: Two numbers --> 6 and 1.  $6(1) = (6)$  ✓  $6 + 1 = 7$  ✓

STEP 5: Rewrite the middle term using these numbers --> middle term (7x) ----->  $6x + x$  rewrite

STEP 6:  $2x^2 + 6x + x + 3$  --> factor by grouping -->  $(2x^2 + 6x) + (x + 3)$  -->  $2x(x + 3) + (x + 3)$

STEP 7: Common binomial  $(x + 3)$  ----->  $(x + 3)(2x + 1)$

**Quick Tips**

- Always check if there's a GCF for the whole trinomial before starting.
- List all factor pairs of  $a \times c$  to find the correct numbers.
- If grouping doesn't work, try different factor pairs or check your numbers.

**Common Mistakes to Avoid**

- ✗ Forgetting to Multiply  $a \times c$ .
- ✗ Choosing wrong pair of numbers (don't guess!).
- ✗ Forgetting to factor out GCF if all terms have one.
- ✗ Incorrect signs or wrong grouping.

**Additional Resources**

**SKILL #21**

CODE: FT.3

# Completing the Square

**Core Concept**

A method that rewrites any quadratic expression ( $ax^2 + bx + c$ ) into the form  $(x + k)^2 + \text{constant}$ , so solving, graphing, or analyzing becomes easy.

**Example**

Complete the square:  $x^2 + 6x + 2$

STEP 1:  $\rightarrow a = 1, b = 6, c = 2$   $a = 1$

STEP 2:  $\frac{b}{2} = \frac{6}{2} = 3 \rightarrow 3^2 = 9$

STEP 3:  $\rightarrow x^2 + 6x + 2 + 9 - 9$

STEP 4:  $x^2 + 6x + 9 + 2 - 9$

STEP 5:  $(x^2 + 6x + 9) - 7 \rightarrow (x + 3)^2 - 7$

**Example 2: When  $a \neq 1$** 

Complete the square:  $2x^2 + 12x + 1$

STEP 1:  $\rightarrow a = 2, b = 12, c = 1$  factor 2 out

$$2(x^2 + 6x) + 1$$

STEP 2:  $\frac{b}{2} = \frac{6}{2} = 3 \rightarrow 3^2 = 9$

STEP 3:  $\rightarrow 2(x^2 + 6x + 9 - 9) + 1$

STEP 4:  $2(x^2 + 6x + 9) - 18 + 1$  (take  $-9$  out)

STEP 5:  $2(x + 3)^2 - 17$

**When to use it?**

- When solving quadratics by factoring fails.
- To find the vertex (highest / lowest point) of a parabola.

**GULDEN RULE**

STEP 1: Make sure the  $x^2$  coefficient is 1 (divide if needed).

STEP 2: Take half of the  $x$ -coefficient ( $b$ ), then square it.

STEP 3: Add & subtract that square inside the expression.

STEP 4: Rearrange the constants.

STEP 5: Factor the perfect square trinomial.

Must have  $a = 1$  before taking half of  $b$ .

If  $a \neq 1$ , factor it out first!

**Common Mistakes to Avoid**

✗ Forgetting to balance

Whatever you add inside the square, also subtract

✗ Leaving  $x^2$  coefficient  $\neq 1$

Always factor it out first.

✗ Wrong Half-and-Square:

For  $x^2 + 8x$ , it's  $\left(\frac{8}{2}\right)^2 = 16$ , not  $8^2 = 64$ .

Always halve first, then square.

**Additional Resources**

**SKILL #26**

CODE: PN.3

# Multiplying Polynomials

 **Core Concept**

When you multiply expressions, you're combining terms using the distributive property and exponent rules. You'll often apply these three main ideas:

- Multiply coefficients (the numbers).
- Apply laws of exponents (add powers of like bases). --> **SKILL #27**
- Use distribution to expand expressions.

**Why It Matters**

These skills help you solve equations, factor expressions, and understand patterns in math, science, and engineering.

**GULDEN RULE**

- Multiply coefficients (numbers) and add exponents for like variables
- Use the distributive property:  $a(b + c) = ab + ac$
- Combine like terms at the end
- Recognize and use special product patterns 🖱️

**Special Products:**

Difference of Squares:

$$(a + b)(a - b) = a^2 - b^2$$

Perfect Square Trinomial:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

**Examples**

Example 1: Multiplying Monomials

$$3x^2 \times 4x^3$$

STEP 1: Multiply coefficients -->  $3 \times 4 = 12$

STEP 2: Apply laws of exponents to multiply  $x^2$  and  $x^3$ :

$$x^2 \cdot x^3 = x^{2+3} \rightarrow x^5$$

STEP 3: Combine -->  $12x^5$  ✅

Example 3: Special Product—Difference of Squares

$$(x + 7)(x - 7)$$

Use the special product rule: -->  $(x + 7)(x - 7) = x^2 - 49$  ✅

Example 2: Multiplying a Monomial by a Polynomial

$$2x \cdot (x^2 + 3x + 4)$$

Use the distributive property:

$$2x \cdot x^2 + 2x \cdot 3x + 2x \cdot 4 \rightarrow 2x^3 + 6x^2 + 8x$$

Example 4: Multiplying Two Binomials (FOIL)

$$(x + 2)(x + 5)$$

First -->  $x \cdot x = x^2$  , Outer -->  $x \cdot 5 = 5x$

Inner -->  $2 \cdot x = 2x$  , Last -->  $2 \cdot 5 = 10$

Combine:  $x^2 + 5x + 2x + 10 \rightarrow x^2 + 7x + 10$

 **Common Mistakes to Avoid**

- ❌ Forgetting to multiply every term in one polynomial by every term in the other
- ❌ Not combining like terms after multiplying
- ❌ Mixing up signs (positive/negative)
- ❌ Forgetting special product patterns

 **Additional Resources**

<https://sanwaralkmali.github.io/>

## SKILL #27

CODE: EP.1

# Laws of Exponents



### Core Concept

An exponent tells you how many times to multiply a base number by itself.

The laws of exponents are rules that help you simplify expressions with exponents.

Example:  $2^4 = 2 \times 2 \times 2 \times 2 = 16 \rightarrow 2^4 = 16$

### Why It Matters

- ❖ Simplify complex algebraic expressions quickly
- ❖ Work with scientific notation
- ❖ Prepare for logarithms and advanced mathematics
- ❖ Solve equations involving powers efficiently

### Key Laws of Exponents

1. Product Rule:  $a^m \times a^n = a^{m+n}$
2. Quotient Rule:  $a^m \div a^n = a^{m-n}$
3. Power Rule:  $(a^m)^n = a^{m \cdot n}$
4. Zero Exponent:  $a^0 = 1$  ( $a \neq 0$ )
5. Negative Exponent:  $a^{-n} = \frac{1}{a^n}$
6. Distribute Powers:  $(ab)^n = a^n \cdot b^n$ ,  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

### Examples

Example 1:  $x^3 \cdot x^5 \rightarrow x^{3+5} = x^8$  ✓

Example 2:  $\frac{x^3}{x^5} \rightarrow x^{3-5} = x^{-2}$  ✓

Example 3:  $(x^2)^3 \rightarrow x^{2 \cdot 3} = x^6$  ✓

Example 4:  $(9x)^0 \rightarrow 1$  ✓

Example 5:  $x^{-3} \rightarrow \frac{1}{x^3}$  ✓

Example 6:  $(3x)^3 \rightarrow 3^3 \cdot x^3 = 27x^3$  ✓

### Extra Practice

Simplify:  $(3x^2y)^3 \times (x^{-3}y^{-2})$

STEP 1: Use the Distribute Powers RULE

$$(3x^2y)^3 \rightarrow 3^3(x^2)^3(y^3) \rightarrow 27x^6y^3$$

STEP 2: Multiply and add exponents

$$27(x^6 \cdot x^{-3})(y^3 \cdot y^{-2}) \\ = 27x^3y$$



### Common Mistakes to Avoid

- ✗ Thinking  $2^4 = 2 \times 4$  (Nope! It's  $2 \times 2 \times 2 \times 2$ )
- ✗ Forgetting that a zero exponent means 1
- ✗ Adding exponents when multiplying different bases  
(Only works with the same base!)



### Additional Resources



Exponents are a shortcut for **repeated multiplication**.

The laws of exponents make it easy to work with big numbers and simplify expressions in algebra.